

Théorie des ensembles

Modes de génération d'un ensemble (Exercices)

Prop.

$$\textcircled{1} A = \{(x, y) \in \mathbb{R}^2 \mid \underline{-2x + 5y + 1 = 0}\}$$

$$B = \{(\underline{-2 - 10t}, \underline{-1 - 4t}) \mid t \in \mathbb{R}\}$$

Montrer $A = B$

ACB Soit $(x, y) \in A$ -2x + 5y + 1 = 0 $\left(x = \frac{5y+1}{2} ; y = \frac{2x-1}{5} \right)$

$(x, y) = (\underline{-2 - 10t}, \underline{-1 - 4t})$ $x = -2 - 10t \Leftrightarrow \boxed{\frac{-x-2}{10} = t}$

$-1 - 4t = -1 - 4 \left(\frac{-x-2}{10} \right) = -1 + \frac{2x+4}{5} = -\frac{5}{5} + \frac{2x+4}{5} = \frac{2x-1}{5} = y$

$\underbrace{(x, y)}_{\in B} \quad \quad \quad \text{ACB}$

BCA Soit $(x, y) \in B$ $x = -2 - 10t$ et $y = -1 - 4t$ pour $t \in \mathbb{R}$

$$\begin{aligned} -2x + 5y + 1 &= -2(-2 - 10t) + 5(-1 - 4t) + 1 \\ &= +4 + \cancel{20t} - 5 - \cancel{20t} + 1 = 0 \end{aligned} \quad (x, y) \in A$$

BCA

Concl. $A = B$

② 1) a) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

C Soit $(x, y) \in (A \cup B) \times C$ $x \in A \cup B$ et $y \in C$

$\left(\begin{array}{l} \text{Si } x \in A, (x, y) \in A \times C \text{ donc } (x, y) \in (A \times C) \cup (B \times C) \\ \text{Si } x \notin A, x \in B, (x, y) \in B \times C \end{array} \right. \quad "$

$\supset$ Soit $(x, y) \in (A \times C) \cup (B \times C)$

$\left(\begin{array}{l} \text{Si } (x, y) \in A \times C \quad \begin{array}{l} x \in A, y \in C \\ x \in A \cup B \end{array} \end{array} \right) (x, y) \in (A \cup B) \times C$

$\left(\begin{array}{l} \text{Si } (x, y) \notin A \times C, (x, y) \in B \times C \quad \begin{array}{l} x \in B, y \in C \\ x \in A \cup B \end{array} \end{array} \right) \text{ idem}$

cd $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$$b) (A \setminus B) \times C = (A \times C) \setminus (B \times C)$$

$$C \quad (x, y) \in (A \setminus B) \times C \quad \begin{array}{c} x \in A \setminus B \\ \underline{x \in A}, \underline{x \notin B} \end{array} \quad \underline{y \in C} \quad (x, y) \in A \times C$$

Si $(x, y) \in B \times C$, alors $x \in B$, absurde $(x, y) \notin B \times C$

$$(x, y) \in (A \times C) \setminus (B \times C)$$

$$\Rightarrow (x, y) \in (A \times C) \setminus (B \times C) \quad (x, y) \in A \times C \quad \begin{array}{c} x \in A \quad \underline{y \in C} \end{array}$$

$$\underline{(x, y) \notin B \times C} \Leftrightarrow \text{NON}(x \in B \text{ ET } y \in C) \Leftrightarrow \begin{array}{c} \underline{x \notin B} \text{ ou } \underline{y \notin C} \\ \text{Vrai} \quad \quad \quad \text{F} \end{array}$$

$$\underline{x \in A \setminus B}$$

$$(x, y) \in (A \setminus B) \times C$$

$$\underline{\text{cd}} \quad (A \setminus B) \times C = (A \times C) \setminus (B \times C)$$

2) Montrer que $(A \times B) \cup (C \times D) \neq (A \cup C) \times (B \cup D)$

$$A = \{1, 2\} \quad B = \{3, 4\} \quad C = \{5, 6\} \quad D = \{7, 8\}$$

$$\left(\begin{array}{l} A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\} \\ C \times D = \{(5, 7), (5, 8), (6, 7), (6, 8)\} \end{array} \right) (A \times B) \cup (C \times D)$$

$$A \cup C = \{1, 2, 5, 6\} \quad B \cup D = \{3, 4, 7, 8\}$$

$$(1, 7) \in (A \cup C) \times (B \cup D)$$

$$(1, 7) \notin (A \times B) \cup (C \times D)$$

Exercice bonus

$$\text{Soit } E = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$$

Montrer que E ne peut pas s'écrire comme le produit cartésien de deux ensembles.

Solutions : martinrousard.fr