Trigonométrie (Exercices)

$$e^{i(a+b)} = e^{ia} \times e^{ib} = (\cos a + i \sin a)(\cos b + i \sin b)$$

1 Formules d'addition et de duplication pour la tangente ?

$$\tan(a+b) = \frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b}$$

$$= \frac{\frac{\sin a}{\cos a} + \frac{\sin b}{\cos b}}{1 - \frac{\sin a}{\cos a}\frac{\sin b}{\cos b}}$$

$$= \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(2a) = \frac{2 \tan a}{1 - \tan^2 a}$$

② Calculer $\cos\left(\frac{\pi}{8}\right)$

$\cos\left(\frac{1}{2}a\right)$?

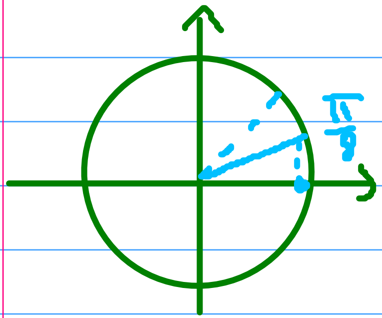
$$\cos(2a) = 2\cos^2 a - 1$$

$$\cos(a) = 2\cos^{1}\left(\frac{a}{2}\right) - 1$$

$$\Leftrightarrow \cos(a) + 1 = 2\cos^2\left(\frac{a}{2}\right)$$

$$\Leftrightarrow \cos^2\left(\frac{a}{2}\right) = \frac{\cos(a)+1}{2}$$

$$a = \frac{\pi}{4} \qquad \cos^2\left(\frac{\pi}{8}\right) = \frac{\cos\left(\frac{\pi}{4}\right)+1}{2} = \frac{\frac{\sqrt{2}}{2}+1}{2} = \frac{\sqrt{2}+2}{4}$$



$$\cos\frac{\pi}{8} > 0$$

$$\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{\sqrt{2}+2}}{2}$$

③ Soit $x \in \mathbb{R}$, exprimer $\cos(3x)$ en fonction de $\cos x$

$$
\begin{aligned}
\cos(3x) &= \cos(2x+x) && \text{Add. } \cos(a+b) \\
&= \cos(2x)\cos x - \sin(2x)\sin x && \text{Dupl.} \\
&= (\cos^2 x - \sin^2 x)\cos x - (2\sin x \cos x)\sin x \\
&= \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x \\
&= \cos^3 x - 3\sin^2 x \cos x && \cos^2 + \sin^2 = 1 \\
&= \cos^3 x - 3(1-\cos^2 x)\cos x \\
&= \cos^3 x - 3\cos x + 3\cos^3 x \\
&= 4\cos^3 x - 3\cos x
\end{aligned}
$$

Exercice bonus

Montrer que $\forall n \geqslant 2$,

$$\cos\left(\frac{\pi}{2^n}\right) = \frac{\overbrace{\sqrt{2+\sqrt{2+\sqrt{2+\dots+\sqrt{2}}}}}^{(n-1)\text{ termes}}}{2}$$

$n=2$ $\quad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\qquad n=3$ $\quad \cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$

$$\cos\left(\frac{\pi}{16}\right) = \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}$$

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