J-S1

Calcul des limites (Exercices)

Calculer les limites :

+∞ a) $\underbrace{e^{2x}}_{\to+\infty} - \underbrace{x\sqrt{x}e^{x}}_{+\infty} + \underbrace{x^4}_{+\infty} = \underbrace{e^{2x}}_{\to+\infty}\underbrace{\left(1 - \underbrace{\frac{x^{3/2}}{e^{x}}}_{\to 0} + \underbrace{\frac{x^4}{e^{2x}}}_{\to 0}\right)}_{\to 1} \xrightarrow[x\to+\infty]{} +\infty$

$e^{x}\times e^{x}$

⚠ Par croissances comparées ...

Par somme ...

Par produit ...

+∞ b) $\underbrace{\sqrt{x+\sqrt{x}}}_{+\infty} - \underbrace{\sqrt{x}}_{+\infty} = \frac{x+\sqrt{x}-x}{\sqrt{x+\sqrt{x}}+\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}} = \frac{1}{\underbrace{\sqrt{\underbrace{1+\underbrace{\frac{1}{\sqrt{x}}}_{\to 0}}_{\to 1}}+1}_{\to 1}}$

$a-b = \frac{a^2-b^2}{a+b}$

$\sqrt{a}-\sqrt{b} = \frac{a-b}{\sqrt{a}+\sqrt{b}}$

$\xrightarrow[x\to+\infty]{} \frac{1}{2}$

Par quotient...

somme ...

composition ...

$+\infty$ c) $$\frac{x\ln x-\sqrt{x}}{x\sqrt{x}+2x}=\frac{\cancel{x}\ln x\overbrace{\left(1-\frac{1}{\sqrt{x}\ln x}\right)}^{\to 1}}{\cancel{x}\sqrt{x}\underbrace{\left(1-\frac{2}{\sqrt{x}}\right)}_{\to 1}}\xrightarrow[x\to+\infty]{}0$$

$$\frac{\ln x}{\sqrt{x}}\to 0$$

$+\infty$ d) $$\frac{\lfloor\ln(\sqrt{x})\rfloor}{\sqrt{x}}$$

$$x-1<\lfloor x\rfloor\leq x<\lfloor x\rfloor+1$$

$$\ln(\sqrt{x})=\frac{1}{2}\ln x$$

$$\frac{\ln(\sqrt{x})-1}{\sqrt{x}}<\frac{\lfloor\ln(\sqrt{x})\rfloor}{\sqrt{x}}\leq\frac{\ln(\sqrt{x})}{\sqrt{x}}$$

$$\underbrace{\frac{\frac{1}{2}\ln x-1}{\sqrt{x}}}_{\to 0}\leq\xrightarrow[x\to+\infty]{}0\leq\underbrace{\frac{1}{2}\,\frac{\ln x}{\sqrt{x}}}_{\to 0}$$

0 e) $$\frac{1-\cos x}{x}=\underline{\frac{\cos(0)-\cos(x)}{h}}\qquad\lim_{h\to 0}\underline{\frac{\cos(0+h)-\cos(0)}{h}}=\cos'(0)=0$$

$$\xrightarrow[x\to 0]{}0$$

$x > 0\ ,\quad \ln(1+x) > 0$

$-1 \le \sin x \le 1$

$-\frac{1}{\ln(1+x)} \le \frac{\sin x}{\ln(1+x)} \le \frac{1}{\ln(1+x)}$

0 f) $\frac{\boxed{\sin(x)}}{\boxed{\ln(1+x)}}$ $\quad \sin(x) \to 0$, $\quad \ln(1+x) \to 0$

$$= \frac{\frac{\sin x}{x}}{\frac{\ln(1+x)}{x}} = \frac{\boxed{\frac{\sin x - \sin 0}{x}}}{\boxed{\frac{\ln(1+x) - \ln(1)}{x}}} \xrightarrow[x \to 0]{} 1$$

$\frac{\sin x - \sin 0}{x} \longrightarrow \sin'(0) = 1$

$\frac{\ln(1+x) - \ln(1)}{x} \longrightarrow \ln'(1) = 1$

$-\infty$ g) $\frac{x \sin x}{1 + x^2}$ $\quad x < 0$

$-1 \le \sin x \le 1$

$-x \geq x \sin x \geq x$

$$\frac{x}{1+x^2} = \underbrace{\boxed{\frac{x}{x^2}}}_{\to 0} \times \underbrace{\boxed{\frac{1}{\frac{1}{x^2} + 1}}}_{\to 1} \xrightarrow[x \to -\infty]{} 0$$

$$\underbrace{\boxed{-\frac{x}{1+x^2}}}_{\to 0} \geq \frac{x \sin x}{1+x^2} \geq \underbrace{\boxed{\frac{x}{1+x^2}}}_{\to 0}$$

$$\frac{x \sin x}{1+x^2} \xrightarrow[x \to -\infty]{} 0$$

0 h) $\frac{\tan x}{x} = \frac{\tan x - \tan 0}{x} \xrightarrow[x \to 0]{} \tan'(0) = \frac{1}{\cos^2(0)} = 1$

0 i) $$\frac{1}{\tan x} - \frac{1}{\sin x} = \frac{\cos x}{\sin x} - \frac{1}{\sin x} = \frac{\cos x - 1}{\sin x} = \frac{\frac{\cos x - \cos 0}{x}}{\frac{\sin x - \sin 0}{x}}$$

$$\xrightarrow[x \to 0]{} \frac{\cos'(0)}{\sin'(0)} = \frac{0}{1} = 0$$

$+\infty$ j) $$\left(1 + \frac{1}{2^n}\right)^{n^2} = \exp\left(n^2 \ln\left(1 + \frac{1}{2^n}\right)\right)$$

$$= \exp\left(\underbrace{\frac{n^2}{2^n}}_{\substack{\to 0 \\ (c.c)}} \times \underbrace{\frac{\ln\left(1 + \frac{1}{2^n}\right)}{\frac{1}{2^n}}}_{\to 1}\right)$$

$$\xrightarrow[n \to +\infty]{} \exp(0) = \underline{1}$$

a^b

↓

$b \ln a$

↓

$e^{b \ln a}$

$$\frac{\ln(1+?)}{?} \xrightarrow[? \to 0]{} \underline{1}$$

$$\frac{1}{2^n} \xrightarrow[n \to +\infty]{} 0$$

Composition

Exercice bonus

Déterminer la limite des suites :

a) $\sqrt{n} \ln\left(\dfrac{\sqrt{n}+1}{\sqrt{n}-1}\right)$

b) $\dfrac{\lfloor 10^n x \rfloor}{10^n}$ (où $x \in \mathbb{R}$)

c) $\dfrac{\sin e^n + e^{\sin n} + n}{\sqrt{n^2+n}}$

Solutions : martintrousard.fr