Applications (Exercices)

① Soit $f: E \to F$ $A_1, A_2 \subset E$ $B_1, B_2 \subset F$ Montrer que :

a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

$(\subset)$ $y \in f(A_1 \cup A_2)$ $\exists x \in A_1 \cup A_2 : y = f(x)$

Si $x \in A_1$, $f(x) \in f(A_1)$ $y \in f(A_1)$

$x \notin A_1$, $x \in A_2$ $f(x) \in f(A_2)$ $y \in f(A_2)$

$y \in f(A_1) \cup f(A_2)$

$(\supset)$ $y \in f(A_1) \cup f(A_2)$

Si $y \in f(A_1)$, alors $\exists x \in A_1 : y = f(x)$ $x \in A_1 \cup A_2$

$y \notin f(A_1)$, $y \in f(A_2)$ $\exists x \in A_2 : y = f(x)$ $x \in A_1 \cup A_2$

$y \in f(A_1 \cup A_2)$

b) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$ $\qquad$ $f^{-1}(B) = \{x \in E \mid f(x) \in B\}$

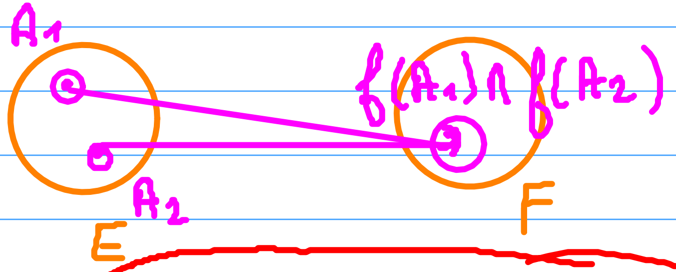
$(\subset)$ $x \in f^{-1}(B_1 \cup B_2)$ $\qquad$ $f(x) \in B_1 \cup B_2$

Si $f(x) \in B_1$, $x \in f^{-1}(B_1)$
$f(x) \notin B_1$, $f(x) \in B_2$, $x \in f^{-1}(B_2)$ $\qquad$ $x \in f^{-1}(B_1) \cup f^{-1}(B_2)$

$(\supset)$ $x \in f^{-1}(B_1) \cup f^{-1}(B_2)$

Si $x \in f^{-1}(B_1)$, $f(x) \in B_1$
$x \notin f^{-1}(B_1)$, $x \in f^{-1}(B_2)$, $f(x) \in B_2$ $\qquad$ $f(x) \in B_1 \cup B_2$
$x \in f^{-1}(B_1 \cup B_2)$

c) $f(\underbrace{A_1 \cap A_2}_{=\emptyset}) \neq f(A_1) \cap f(A_2)$



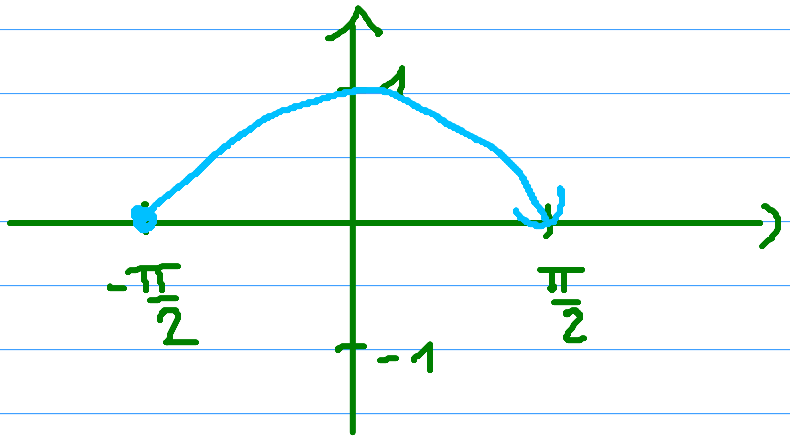
$f : x \mapsto x^2$ $\qquad$ $A_1 = \{1\}$ $\qquad$ $A_2 = \{-1\}$

$A_1 \cap A_2 = \emptyset$ $\quad$ $f(A_1 \cap A_2) = \emptyset$ $\quad$ $f(A_1) = \{1\}$ $\quad$ $f(A_2) = \{1\}$ $\quad$ $f(A_1) \cap f(A_2) = \{1\}$

② Déterminer :

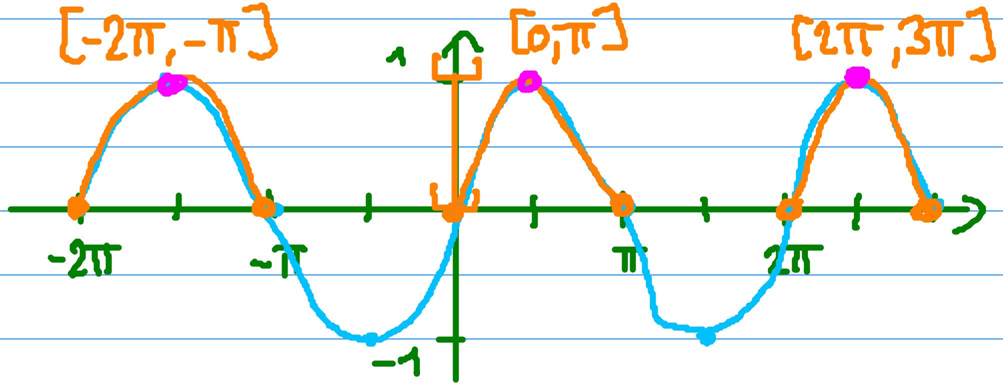
a) $\cos\left(\left[-\frac{\pi}{2}, \frac{\pi}{2}\right[\right)$

$= [0,1]$

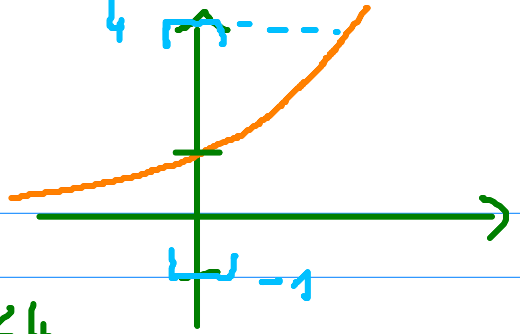


b) $\sin^{-1}([0,1[)$

$$\bigcup_{k\in\mathbb{Z}}\left([2k\pi,(2k+1)\pi] \setminus \left\{2k\pi+\frac{\pi}{2}\right\}\right)$$



c) $\exp^{-1}([-1,4]) =]-\infty, \ln 4]$



$x \in \exp^{-1}([-1,4]) \iff \exp(x) \in [-1,4] \iff \exp(x) \le 4$

$\iff x \le \ln(4)$

d) $f(\mathbb{R})$ ou $f : x \longmapsto \dfrac{x}{1+x^2}$ $\qquad \lim\limits_{x \to \pm\infty} f(x) = 0$

$$f'(x) = \frac{1 \times (1+x^2) - x(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} = \frac{(1-x)(1+x)}{(1+x^2)^2}$$

x	$-\infty$		-1		1		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
f	0	$\searrow$	$-\frac{1}{2}$	$\nearrow$	$\frac{1}{2}$	$\searrow$	0

$f(\mathbb{R}) = \left[-\frac{1}{2}, \frac{1}{2}\right]$

Exercice bonus

Soit $f : \begin{cases} \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \\ (a,b) \longmapsto (a+b, ab) \end{cases}$. Déterminer $f(\mathbb{R}^2)$.

$(c,d) \in f(\mathbb{R}^2)$ $\quad \exists (a,b) \in \mathbb{R}^2 : \begin{cases} a+b = c \\ ab = d \end{cases}$

$\Leftrightarrow$ a et b solut° de $x^2 - cx + d = 0$

Conditions ?

Solutions : martintroussard.fr