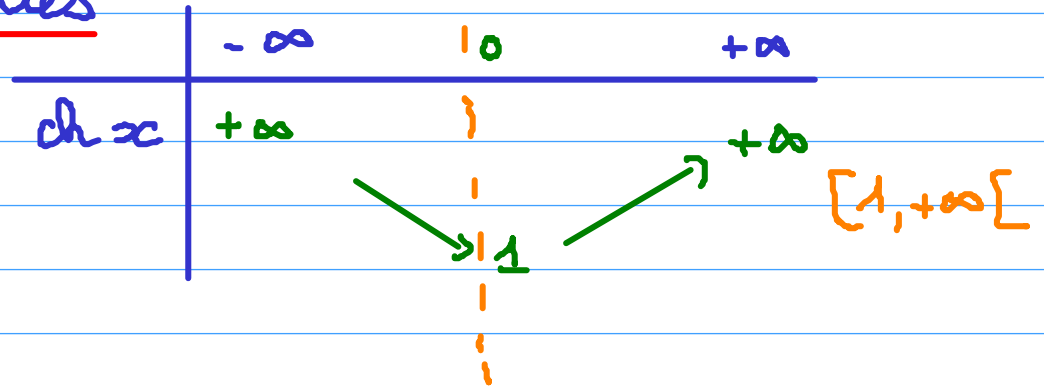


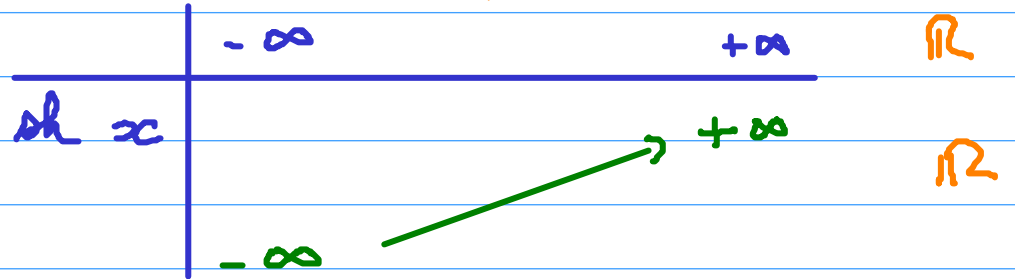
Fonctions hyperboliques réciproques

Rappel : Fonctions hyperboliques

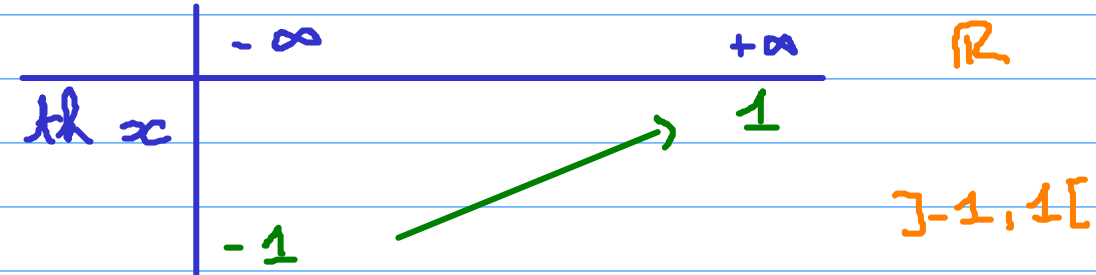
$$\operatorname{ch} : x \mapsto \frac{e^x + e^{-x}}{2}$$



$$\operatorname{sh} : x \mapsto \frac{e^x - e^{-x}}{2}$$



$$\operatorname{th} : x \mapsto \frac{\operatorname{sh} x}{\operatorname{ch} x}$$



acoh

Argch $\overset{f}{\text{ch}}: \mathbb{R}_+ \rightarrow [1, +\infty[$ bijective $\overset{f^{-1}}{\text{argch}}: [1, +\infty[\rightarrow \mathbb{R}_+$

f dérivable en a $\Rightarrow f^{-1}$ dérivable en $b = f(a)$ et $f'(a) \neq 0$ $a = f^{-1}(b)$ et $f'(b) = \frac{1}{f'(a)} = \frac{1}{f'(f^{-1}(b))}$

$$\text{ch}'(a) = 0 \Leftrightarrow \text{sh}(a) = 0 \Leftrightarrow a = 0 \quad b = \text{ch}(a) = 1$$

$$\text{argch} \text{ défini sur }]1, +\infty[\text{ et } \forall y > 1, \text{argch}'(y) = \frac{1}{\text{sh}(\text{argch}(y))}$$

$$\text{ch}^2 \dots - \text{sh}^2 \dots = 1 \Leftrightarrow \text{sh}^2 \dots = \text{ch}^2 \dots - 1 \quad \text{argch}(y) > 0 \Rightarrow \text{sh}(\text{argch}(y)) > 0$$

$$\Rightarrow \text{sh}(\text{argch}(y)) = \sqrt{\text{sh}^2(\text{argch}(y))} = \sqrt{\text{ch}^2(\text{argch}(y)) - 1} = \sqrt{y^2 - 1}$$

$$\boxed{\text{argch}'(y) = \frac{1}{\sqrt{y^2 - 1}}}$$

$$\arccos'(y) = - \frac{1}{\sqrt{1 - y^2}}$$

Argsh ^b sh: $\mathbb{R} \rightarrow \mathbb{R}$ bijective

⁻¹ argh: $\mathbb{R} \rightarrow \mathbb{R}$

$$sh'(a) = 0 \Leftrightarrow ch(a) = 0 \Leftrightarrow a \in \emptyset$$

argsh dér. | $\mathbb{R}$

$$argsh'(y) = \frac{1}{ch(argsh(y))} = \frac{1}{\sqrt{ch^2(\dots)}} = \frac{1}{\sqrt{1+sh^2(argsh y)}} = \frac{1}{\sqrt{1+y^2}}$$

$$arcsin'(y) = \frac{1}{\sqrt{1-x^2}}$$

Argth th: $\mathbb{R} \rightarrow]-1, 1[$ bijective

argth: $] -1, 1[\rightarrow \mathbb{R}$

$$th'(a) = 0 \Leftrightarrow \frac{1}{ch^2(a)} = 0 \Leftrightarrow a \in \emptyset$$

arg th dér | $] -1, 1[$

$$argth(y) = \frac{1}{th'(argth y)} = \frac{1}{1-th^2(argth y)} = \frac{1}{1-y^2}$$

$$arctan'(y) = \frac{1}{1+y^2}$$

Expressions logarithmiques

$$\operatorname{argch}: [1; +\infty[\rightarrow \mathbb{R}_+$$

$$y \in [1, +\infty[$$

$$\underline{x = \operatorname{argch}(y)} \Leftrightarrow y = \operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$x \geq 0$

$$\Leftrightarrow e^x - 2y + e^{-x} = 0$$

$$\Leftrightarrow e^{2x} - 2ye^x + 1 = 0$$

$$\Leftrightarrow X^2 - 2yX + 1 = 0 \quad \text{ou } X = e^x$$

$$\Delta = 4y^2 - 4 = 4(y^2 - 1) \geq 0$$

$$X = \frac{2y \pm 2\sqrt{y^2 - 1}}{2} = y \pm \sqrt{y^2 - 1}$$

$$x \geq 0 \quad e^x = X \geq 1 \quad \text{et} \quad \underline{y - \sqrt{y^2 - 1} \leq 1} \quad X = y + \sqrt{y^2 - 1}$$

$$x = \operatorname{argch}(y) = \ln(y + \sqrt{y^2 - 1})$$

Méthode par dérivation - intégration

$$\operatorname{argth} :]-1, 1[\rightarrow \mathbb{R}$$

$$\operatorname{argth}'(x) = \frac{1 + 0x}{1 - x^2} = \frac{1}{(1-x)(1+x)} = \frac{a}{1-x} + \frac{b}{1+x} = \frac{(a-b)x + (a+b)}{1-x^2}$$

$$= \frac{1}{2} \left(\frac{1}{1-x} + \frac{1}{1+x} \right)$$

$$\frac{1}{1+x} \rightarrow \ln|1+x| = \ln(1+x)$$

$$\frac{1}{1-x} \rightarrow -\ln|1-x| = -\ln(1-x)$$

$$\operatorname{argth}(x) = \frac{1}{2} (-\ln(1-x) + \ln(1+x)) + C = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) + C$$

$$\operatorname{argth}(0) = 0 \Rightarrow C = 0$$

$$\boxed{\operatorname{argth}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)}$$

À vous de jouer!

① Résoudre dans $\mathbb{R}$:

$$\operatorname{argch} x + \operatorname{argsh} x = 1$$

② Simplifier :

a) $\operatorname{argch} \left(\sqrt{\frac{1+x}{2}} \right)$

b) $\operatorname{argth} \left(\frac{1+2\operatorname{th} x}{2+\operatorname{th} x} \right)$