

Inversion de matrices (Exercices)

① Inversen

$$a) A = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} \quad \begin{pmatrix} + & - \\ - & + \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} {}^t C \quad \det(A) = -1$$

$$\text{Comatrice } C = \begin{pmatrix} -1 & -3 \\ -2 & -5 \end{pmatrix} \quad {}^t C = \begin{pmatrix} -1 & -2 \\ -3 & -5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} {}^t C = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$$

$$AA^{-1} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \checkmark$$

$$b) A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 2 & 7 & 7 \end{pmatrix} \quad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \begin{matrix} \nearrow \text{det.} \\ \searrow \text{comatrice} \end{matrix}$$

$$\det A = +1 \times \underbrace{(4 \times 7 - 7 \times 3)}_7 - 3 \times \underbrace{(1 \times 7 - 2 \times 3)}_1 + 3 \times \underbrace{(1 \times 7 - 2 \times 4)}_{-1} = 1$$

$$\text{Comatrice } C = \begin{pmatrix} 7 & -1 & -1 \\ 0 & 1 & -1 \\ -3 & 0 & 1 \end{pmatrix} \quad A^{-1} = \frac{1}{\det A} {}^t C = \begin{pmatrix} 7 & 0 & -3 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$$

$$AA^{-1} = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 2 & 7 & 7 \end{pmatrix} \begin{pmatrix} 7 & 0 & -3 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \checkmark$$

② Déterminer $x \in \mathbb{R}$ tel que $A = \begin{pmatrix} x & x+1 & x+2 \\ x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \end{pmatrix}$ soit inversible.

$$\det(A) = x \times [\underline{(x+2)(x+4)} - (x+3)^2]$$

$x = x+3$
 $(x-1)(x+1) = x^2 - 1$
 $= (x+3)^2 - 1$

$$- (x+1) \times [\underline{(x+1)(x+4)} - (x+2)(x+3)]$$

$$+ (x+2) \times [\underline{(x+1)(x+3)} - (x+2)^2]$$

$x = x+2$

$$= x \times (-1) - (x+1) \times (-2) + (x+2) \times (-1)$$

$$= -x + 2x + 2 - x - 2$$

$$\det(A) = 0$$

A n'est jamais inversible

Exercice bonus

Pour quelles valeurs $m \in \mathbb{R}$ la matrice

$$\begin{pmatrix} m & 1 & 1 \\ 1 & m & 1 \\ 1 & 1 & m \end{pmatrix}$$

est-elle inversible ?

Solutions : martintraussard.fr