

Matrices et systèmes d'équations (Exercices)

① Résoudre

$$\begin{cases} -x + 2y + z - t = 2 \\ x - y - z + 2t = 3 \\ -x + y + z + t = 4 \\ -2x + 5y + z - 2t = 6 \end{cases}$$

$$\times(-1) \left(\begin{array}{cccc|c} +1 & -2 & -1 & +1 & -2 \\ 1 & -1 & -1 & 2 & 3 \\ -1 & 1 & 1 & 1 & 4 \\ -2 & 5 & 1 & -2 & 6 \end{array} \right)$$

$$\begin{cases} x - 2y - z + t = -2 \\ y + t = 5 \\ 3t = 7 \\ -z - t = -3 \end{cases}$$

$$\begin{array}{l} -L_1 \\ +L_1 \\ +2L_1 \end{array} \left(\begin{array}{cccc|c} 1 & -2 & -1 & 1 & -2 \\ 0 & 1 & 0 & 1 & 5 \\ 0 & -1 & 0 & 2 & 2 \\ 0 & 1 & -1 & 0 & 2 \end{array} \right)$$

$$t = \frac{7}{3}$$

$$y = 5 - t = \frac{8}{3}$$

$$z = 3 - t = \frac{2}{3}$$

$$x = -2 + 2y + z - t = \frac{5}{3}$$

$$\begin{array}{l} +L_2 \\ -L_2 \end{array} \left(\begin{array}{cccc|c} 1 & -2 & -1 & 1 & -2 \\ 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 3 & 7 \\ 0 & 0 & -1 & -1 & -3 \end{array} \right)$$

② Inverse

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ -6 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \textcircled{1} & 1 & 2 & | & 1 & 0 & 0 \\ 2 & 0 & -1 & | & 0 & 1 & 0 \\ -6 & -1 & 0 & | & 0 & 0 & 1 \end{pmatrix}$$

$$\times (-2) \begin{pmatrix} 1 & 0 & -1/2 & | & 0 & 1/2 & 0 \\ 0 & 1 & 5/2 & | & 1 & -1/2 & 0 \\ 0 & 0 & \textcircled{1} & | & -2 & -5 & -2 \end{pmatrix}$$

$$\begin{matrix} -2L_1 \\ +6L_1 \end{matrix} \begin{pmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 0 & -2 & -5 & | & -2 & 1 & 0 \\ 0 & 5 & 12 & | & 6 & 0 & 1 \end{pmatrix}$$

$$\begin{matrix} +\frac{1}{2}L_3 \\ -\frac{5}{2}L_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & | & -1 & -2 & -1 \\ 0 & 1 & 0 & | & 6 & 12 & 5 \\ 0 & 0 & 1 & | & -2 & -5 & -2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -2 & -1 \\ 6 & 12 & 5 \\ -2 & -5 & -2 \end{pmatrix}$$

inverse

$$\div (-2) \begin{pmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 0 & \textcircled{1} & 5/2 & | & 1 & -1/2 & 0 \\ 0 & 5 & 12 & | & 6 & 0 & 1 \end{pmatrix}$$

$$\begin{matrix} -L_2 \\ -5L_2 \end{matrix} \begin{pmatrix} 1 & 0 & -1/2 & | & 0 & 1/2 & 0 \\ 0 & 1 & 5/2 & | & 1 & -1/2 & 0 \\ 0 & 0 & -1/2 & | & 1 & 5/2 & 1 \end{pmatrix}$$

Exercice bonus

Soit $m \in \mathbb{R}$, résoudre

$$\begin{cases} x + y + mz = 1 \\ x + my + z = 1 \\ mx + y + z = 1 \end{cases}$$

Solutions : martintraussard.fr